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by Alexander H. Kenigsberg, M. ASCE

STRUCTURAL DIVISION

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INTRODUCTION TO SEMI-RIGID DETERMINATE POLYGONAL TRUSSES

Alexander H. Kenigsberg,* M. ASCE

SYNOPSIS

Novel types of truss structures having rigid polygonal web systems are presented.**

In this Introduction, semi-rigid pentagonal trusses only are dealt with at length, setting forth:

1) The basic concepts, among which are definition and premise for rendering them statically determinate;
2) A general procedure for stress analysis;
3) A numerical design procedure, including truss deflections; and,
4) Some distinguishing characteristics.

In the three appendices there are given:

1) An alternate treatment of end panels;
2) An introduction to shallow type trusses; and,
3) A comparison of estimated weights for pentagonal and conventional trusses.

INTRODUCTION

Notwithstanding the fundamental developments in steel design and construction brought about by the advance in the art of arc welding, triangular frame trusses have remained quite immune to changes. Since the principles and merits of rigid framing have universally been acclaimed and widely adopted in both masonry and steel construction, the laggardness in adapting them to truss framing can be attributed only to the realization that there has been little to gain in developing continuity by extending arc welding to ordinary steel trusses. Simple and direct in theory, though inexact in application, conventional riveted trusses have thus remained, basically, axially stressed frame structures in which, as compared to truly rigid frames, rigidity and stability are often limited, primarily because of slender web compression members.

Quadrangular panel, rigid type trusses have been proposed and built. However, they have not become firmly rooted, especially in bridge construction, partly because of unrealistic and confining design assumptions and complex mathematical procedures, and partly because of lack of rigidity and economy, even though often superior in esthetical appeal.

*Civ. and Structural Engr., Nashville Dist., Corps of Engrs., Nashville, Tenn.
**Patents for these trusses have been applied for and are pending.

The polygonal trusses presented here are believed to overcome some of the fundamental limitations inherent in both conventional and quadrangular trusses, yet combining desirable analytical and structural characteristics of the former with the structural behavior of rigid frames. In general, the new type trusses are characterized by fixed joints introduced midheight between the chords, whereby geometrically the web systems are composed of combinations of triangles with four- and five- or six-sided polygons. In this manner objectionably long web members are eliminated; vertical truss shears are carried primarily by the web system in axial stress, as in ordinary trusses, and longitudinal shears in flexural stress thereby imparting true intrinsic rigidity to the entire truss structure.

The combinations of triangles with quadrangles, pentagons and hexagons are many and varied. Only a few are indicated in the paper. Thus, Figure 1 illustrates basic pentagonal trusses, with a variable panel length through truss in (a) and a deck type truss in (b); Figure 2, in (a) and (b), shows basic hexagonal trusses; Figure 3, in (a) and (b) respectively, illustrates bridge and roof type trusses which are modified pentagonal trusses having subdivided panels; while Figure 4 illustrates a double stem pentagonal truss. Figure 5(a) shows the pentagonal bridge truss chosen for detailed analysis, while the accompanying Figures 5(b) and (c) and Figure 6 are introduced for comparison and are treated separately later.

It can be observed that the several types of polygonal trusses shown are, broadly speaking, mutations of the primary pentagonal web trusses of Figure 1. Accordingly, in this Introduction the discussion is confined primarily to the most elementary type of polygonal trusses, namely, the semi-rigid pentagonal truss which is composed of triangles and pentagons. In this form it is determinate analytically and is generally representative of the fundamental characteristics of all polygonal trusses.

Basic Concepts

Basic semi-rigid polygonal trusses are defined as frame structures composed of appropriately grouped combinations of trigonal and pentagonal frames having fixed inside and hinged outside joints. To maintain this pattern the end panels preferably are quadrangular frames; however, it will be shown that the latter can be readily converted into triangular frames, sometimes with worthwhile beneficial results.

These semi-rigid trusses become simple, statically determinate structures when there is applied to them the axiomatic concept that moments at a rigid joint are absorbed by its members in proportion to their stiffness. Thus, at each inside fixed joint of the basic pentagonal truss (a) or (b) of Figure 1, the moment induced in the stem of the Y by the differential horizontal stress in its hinged bottom chord members is resisted by its arms whose far joints also are hinged.

The special approach, of fixed inside and hinged outside joints, is adopted for this paper because it provides a simple, direct and uniform analytical treatment for polygonal trusses in general. Thus hexagonal and double stem pentagonal trusses become "statically determinate" structures if the realistic and simplifying assumption is made that points of contraflexure exist midway along the vertical and horizontal web members respectively.

It is obvious that polygonal trusses can be rendered structurally stable by fixing any number of groups of joints, such as those connecting the chord members to the stems of the Ys, or other groups judiciously chosen, as well,

of course, as by fixing all the joints. Since this is contrary to the concern of this paper, such procedures are not considered here. Also, the placing of primary fixed joints at midheight of the truss is fundamental to the concepts advanced here in that it alone affords a practicable uniformity of construction procedure whereby any basic or modified Y-segment becomes the element of fabrication, assembly and erection for all types of polygonal trusses.

The requirement for establishing algebraically the stability of semi-rigid pentagonal trusses can be expressed by an equation similar to the familiar equation for conventional trusses, namely:

$J_f = 2n - 3 - N$, in which,

J_f = minimum required number of fixed joints, being equal to the number of inside web joints;
n = total number of truss joints; and
N = total number of truss members.

Basic Stress Analysis

The procedure for stress determination in semi-rigid polygonal trusses is essentially arithmetical. This is readily established by inspection of the truss proper in any of the figures or of a typical segment, such as that shown in Figure 7(a), and can be summed up thus:

1) Obtain the external hinge stresses by the method of sections through the truss;
2) Write the moment in the stem of the Y at its M-joint and distribute this moment between the arms of the Y in any rational or arbitrary proportion;
3) Apply the resulting shears to both ends of each arm; and
4) Obtain the axial stresses in all the members of the segment by the usual method of resolution (preferably by graphics) of the loads and shears applied at its joints.

Numerical Stress Determination

Figure 5(b) shows an as-built conventional truss which is considered a well designed structure.* It is a 304 foot simple bridge span designed for H15 Loading and in accordance with the A.A.S.H.O. Specifications. For purposes of subsequent comparison, its overall dimensions, panel lengths and panel loads have been adopted for the pentagonal truss of Figure 5(a), which was then designed as a semi-rigid structure conforming to the above specifications. The accompanying stress diagrams, tables of stresses, deflections and weight estimates are based on this "redesign."

The stress analysis of a single panel of the pentagonal truss forms the basic pattern for the entire truss and for all semi-rigid polygonal trusses as a class. Accordingly, there is presented here, by means of Table 1 and several diagrams in Figure 7, a numerical procedure for determination of the maximum stresses in the members of a typical segment, such as segment YM4 of the truss shown in Figure 5(a). Index loads have been employed throughout this procedure.

*Designed by and under the supervision of Mr. T. E. Eubank, Structural Engineer, Corps of Engineers, Nashville, Tennessee.

The bending moments, shears and hinge loads which, in general, determine the stresses in the members of the segment are indicated diagrammatically in Figure 7(a). The index reactions at hinges U3, U5 and L4 due to external loads are obtained first by the usual method of truss sections through these joints and are identified by the horizontal and vertical lines on the stress diagrams of Figure 7. The induced index shear in the vertical M4L4 is the difference between the index stresses in the bottom chord members L4L6 and L2L4 which, of course, is the longitudinal shear in the panel as well as the difference between the H-components of the forces at the hinged U-joints of the segment. The index moment at the M-joint of M4L4 is the product of its index shear and length and is here distributed between the bars U3M4 and M4U5 in proportion of 1.33 to 1.0. The index shears thus induced in the arms of the Y by the moments absorbed at their M4 joints are then applied, as illustrated on the diagram, at their hinged joints U3 and U5 where, in combination with the index reactions, they are used to determine the axial index stresses in all the members of a segment. The procedure may be followed more readily by simultaneous reference to diagrams (a) and (b) of Figure 7 and columns 1, 2, 3, 4 and 5 of the table which jointly show the combination of graphical and analytical methods used for establishing the index stresses in the several members of the segment due to unit panel loads on the entire span.

The accompanying stress diagrams (c), (d) and (e) of Figure 7 are also worked with unit panel loads applied at the L-joints of the truss and placed, as indicated on each diagram, to obtain the maximum index stress for any member of the segment. For brevity their index stresses are not reproduced in Table 1.

The applicable index stresses are converted into actual shears, moments and axial stresses by means of constant multipliers, indicated in the table as K_D, K_L and K_C, which respectively represent the panel dead load, specification uniform load and concentrated live load. These stresses appear in columns 6 to 14 inclusive of the table. Impact and, finally, maximum total stresses, columns 15 to 20 inclusive, are then obtained in the customary manner.

The maximum axial stresses in the chord system are developed when the truss is fully loaded, the same as axial stresses in the chords of ordinary trusses. Flexure is the major stress in the web system and, for any Y segment, is a direct function of the differential of its bottom chord stresses. As indicated by the H-components of diagram 7(c), this differential is a maximum when the uniform live load is applied to the larger portion of the truss including the panel preceding the segment in question. The axial stress in the web members is due primarily to the vertical shears in the truss, similarly to conventional trusses, but constitutes only a minor fraction of their total stress. Accordingly, the stress increment, when the direct stress in the diagonal U3M4 is a maximum due to the loading indicated by stress diagram (d), is of little consequence in design. Since flexure in the web system is determined by the horizontal shears due to the axial stresses in the chord members, the direction of web bending moments is constant throughout the structure. Consequently, the term "stress reversal" has no significance in polygonal trusses. The reversal of the minor axial stress, from compression to tension, in the diagonal M4U5 due to the specification concentrated live load, as shown by stress diagram (e), is negligible and of negative value only in that it indicates no increment in its basically compressive stress. Consequently, the steps represented by diagram (d) for partial loading and

diagram (e) for concentrated loads can be omitted when the dead load on the truss is preponderant, as in the case of the highway bridge under consideration. This is confirmed by an examination of Table 1, which, for brevity, presents here the stresses for only a single segment of the truss. When expanded to show the maximum total stresses for the entire truss, the table reveals that the bending stresses obtained through diagrams (b) and (c) for uniform dead and live loads only, and including impact, are within 98 to 99%, while the axial stresses are within 96 to 97% of the absolute maximum total stresses as obtained by working all the stress diagrams of Figure 7. However, for railway bridges, where both uniform and concentrated live loads are proportionately greater, both stress diagrams (d) and (e) may have to be employed to establish critical design conditions.

It is to be noted that, for the hypothetical "semi-rigid" pentagonal truss treated here, the given tabular moments, and to some extent the axial stresses also, are disproportionately large due to the fact that the web system alone is made to carry the moments developed in the structure. In actual practice polygonal trusses would have rigid joints throughout, whereby a portion of the frame moments would be absorbed by the top and bottom chord systems thus reducing correspondingly the magnitude of the moments here carried to the M-joints. As in any rigid frame structure, the relative magnitude and distribution of flexural stresses in the members of polygonal trusses are a direct function of the stiffness of its members and the rigidity of its joints. Optimum truss rigidity is attained when member stiffness is balanced by joint rigidity.

Design

Only general features of design pertaining to polygonal trusses are dealt with in this section.

All the members of the truss in Figure 5(a), and as shown in greater detail for the typical segment YM4 in Figure 8, are designed as shop weldments that can be assembled in the field by conventional methods. Briefly, the axially stressed top and bottom chords are standard box sections, although for purposes of illustration, the bottom chord in Figure 8 is shown of H-sections since they produce here a relatively simple and effective floor beam-truss connection. The flexurally stressed web members, shown of wide flange sections in that figure, are tapered bars that conform functionally to their predominant stress, both for economy and appearance. Most of the web members require flange cover plates to provide for the substantial bending moments.

It can be shown that the section moduli of a tapered steel girder vary along the length of the girder practically in proportion to its depth. Since the bending moments, which generally determine the web member cross sections, are also a linear function of their length, it is a simple matter to express graphically the relationship between the moments and section moduli of a tapered member and thus determine the required length of its flange cover plates. Figure 9 illustrates the procedure as applied in establishing the cover plate lengths for the bar M4L4 of the YM4 segment. The tensile axial stress, represented by line f_d, varies little and is algebraically combined with the flexural stresses in the member not to exceed the permissible specification stresses. For either tension or compression side of the member, the lines s_o represent the available section moduli in the web and flanges for the tapered H-sections of Figure 8, while lines s_1 and s_2 show, respectively, the additional section moduli supplied by one or two cover plates. The lines s_R represent the required section moduli along the length of the bar after allowance

for the direct stress. Consequently, the intersections of the s_1 and s_2 lines with the s_R lines establish the required theoretical length of cover plates as measured from the fixed M-joint.

The joints of polygonal trusses are necessarily of great concern in design since, in addition to axial stresses, they must be capable of distributing throughout the frame the heavy primary moments induced in the members of the web system. The midheight M-joints are the crux of these trusses in general, and of the truss under review in particular, since they are the only rigid joints in the structure which resist the direct and bending stresses from the three members of the Y. Figure 8 indicates that balanced rigidity of the Y-frame is readily provided by the functional outline of its members. The chord joints, although assumed hinged for the hypothetical truss of the paper, are detailed as they would normally be in an actual truss and are similar to those of conventional trusses but, as stated above, with the added condition that substantial bending stresses are carried through them from the web system.

At the compression chord joints U3 and U5 of Figure 8 there is indicated a butt type joint that is introduced here, without commending, but merely to invite attention to it on account of some of its merits and as a type that is inherent to and feasible with polygonal trusses only. Of course, conventional gussets continuous across the joints can readily be employed, and may be especially preferable when the arms of the Y subtend the tension chord. Since each Y-segment, or an assembly of a group of segments, is independently stable, it becomes possible to erect polygonal trusses by whole sections instead of the bar-by-bar procedure employed in erecting ordinary trusses. The butt type joints seem particularly well adapted for such erection by fully preassembled truss sections, since they are easy to fit up and totally free of stresses resulting from clearance requirements for reentrant fitting at the joints of conventional trusses.

Truss Deflections

Williot diagrams for graphical determination of deflections in conventional trusses are based essentially on the serial construction of aggregations of triangles whose three sides are established in advance. Hence they are inapplicable to polygonal trusses which are composed of combinations of 3-, 4-, and 5- or 6-sided polygons. In consequence, the deflections of the semi-rigid pentagonal truss dealt with in this paper were determined analytically according to the method given in the book on "Stresses in Framed Structures" (1923) by Hool and Kinne, page 360 and following. This algebraic method is based on "equating the external work done by the load supported by the truss to the internal work performed by the members of the truss" and employs a unit dummy load applied at the point and in the direction of the desired deflection.

The procedure for finding the deflection due to dead load axial stresses is given in detail in Table 2 for joint L10 of the pentagonal truss of Figure 5(a). Columns 2, 3 and 4, respectively, show the lengths, areas and dead load axial stresses in the members of the truss. Column 5 represents the strain in each truss member times E/1000, in which E is the elastic modulus for steel taken as 29,000,000 psi. Column 6 shows the axial stress, in pounds, in the truss members due to a downward load of 1 lb. applied at joint L10. Column 7, the product of column 5 and 6, represents the share contributed to the deflection of that joint by each member of the truss. Its summation is the resulting deflection D for joint L10 which, corrected as indicated at the bottom

of the table, equals 3.08 inches. The deflection of the equivalent joint L5 of the conventional truss of Figure 5(b), due to the same dead load and by the same procedure, is equal to 3.38 inches, or about 11% greater than that of the polygonal truss.

A brief study of the table and further comparison with deflection data obtained for the conventional truss reveal the following pertinent facts:

1) The truss deflection due to the strains in the chord systems alone is virtually identical for the two types of trusses, which is to be expected, since the chord stresses and chord lengths are virtually identical; and
2) The truss deflection due to the strains in the web system alone accounts in the pentagonal truss for less than 7% of the total deflection, whereas it is nearly 17% in the equivalent conventional truss, which too is to be expected, since the web members of the new truss are materially shorter in length and of greater cross-sectional area.

Truss Characteristics

In the course of developing the polygonal trusses it was discovered that they are endowed with certain characteristics, analytical and structural, that distinguish them from conventional and quadrangular trusses. Since these are not self-evident from a casual examination of the sketches or perusal of the descriptive matter in the paper it is considered pertinent to this Introduction of novel structures to briefly list some of them in order to provide a broader base for discussion:

1) Although essentially a rigid frame, the basic structural behavior of the semi-rigid pentagonal truss is in accordance with the fundamental concepts of truss action in that the truss loads are carried by its members primarily in axial stress, similarly to conventional trusses. Its behavior as a rigid frame is due exclusively to the fact that the longitudinal shears are translated into flexural stress.
2) By virtue of the intimate interaction of shearing, direct and flexural stresses, the polygonal truss is intrinsically endowed with both initial and reserve strengths that virtually eliminate the possibility of structural failure under any conceivable conditions of loading. This is in conformity with the fundamental concept of the rigid frame that distress in any of its members is instantly relieved by the other members at the joint to the full extent of their collective elastic and ultimate strength.
3) Introduction of rigid joints near the midheight of the truss results in the shortening of the path of stress transfer through an exceptionally stiff and materially shortened web system.
4) By virtue of the shorter web system whose members are characteristically of variable sections and lengths, it is not likely that objectionable harmonic vibrations would ever set up in the truss members through the application of rhythmic loads.
5) The combination of different types of polygons affords a variety for esthetic treatment not available to conventional trusses composed of triangles only. Moreover, in view of the inherent stiffness of the members of the Y-web system, there appears no need for secondary stiffening members which usually mar conventional trusses.
6) By virtue of the rigid joints located between their chords, pentagonal trusses are readily transformed into two-plane structures, and hexagonal trusses into three-plane structures, without in any way affecting

their structural behavior. This important characteristic enables such "bent" trusses to be combined into rigid space frames that can be advantageously adapted to many structures, such as bridges, hulls of ships and airframes.

CONCLUSION

The primary goal here has been the introduction of novel polygonal trusses and to present, by means of an idealized "semi-rigid" pentagonal frame, some of their fundamentals which are believed to combine the desirable characteristics of axially stressed structures with those of rigid frames. In actual construction of polygonal trusses the web-to-chord connections would, of course, be essentially fixed joints. Since analytical treatment alone for all-rigid frames would yield only approximate solutions which may be quite inaccurate for structures yet untried, there is indicated a need for comprehensive experimental investigations into the structural behavior of such polygonal trusses. It is believed that such a procedure would serve to establish reliable and realistic criteria for their design, since a thorough testing program would fully disclose the actual stress distributions throughout these trusses and, especially, points of critical stress concentrations; locations of points of inflexion, particularly in the tapered web members; degree of rigidity and the character of their resistance to distortion, since both flexural and axial stressing are involved; ultimate load carrying capacities and the character of instability under such loading; and other characteristics that flow from the special construction of these trusses.

Of the appendices that follow, A and B deal with alternate end panel arrangements and shallow type trusses respectively, while appendix C presents parallel tables of estimated weights and their distribution for semi-rigid pentagonal trusses versus those of the conventional truss of comparison.

APPENDIX A - ALTERNATE END PANEL ARRANGEMENT

For uniformly loaded trusses the increment in chord stress is a maximum for the end panels. In pentagonal trusses, in which the increment in chord stress is translated into moments, largely in the web system, it may result in uneconomical sections for the web members of the end segment. The arrangement shown in Figure 6 presents an alternate treatment in which an auxiliary diagonal is introduced in the end panel with the result that substantial economy of materials is accomplished thereby.

By referring to diagrams (b) and (c) of Figure 10, in which the alternate scheme is treated analytically, it can be seen that the stresses in all the members of the end panel become axial only, yet retaining the rigidity of the Y formation. The relative merits of the basic and modified arrangements are revealed in detail by Table 3 in which are listed the total maximum stresses for and the estimated weights of the end panel members for the two schemes, assuming hinged joints throughout the panel. The indicated saving in the weight of steel in the end panel as a whole, due to the modified arrangement, is thus well over 20% of that of the basic design.

Introduction of the auxiliary diagonal in the end panel is also an added advantage from the standpoint of safety in cases of emergencies not too infrequent in bridge structures. Thus, when the end post is damaged, from any

cause, the auxiliary diagonal LOM2 will take over the stress from the damaged member, insuring thereby stability of the truss and the structure of which it is a part, until the injured member is restored. That continuity of truss geometry and, therefore, truss action are still maintained in such an emergency can be visualized by mentally removing from the modified end panel of Figure 6 the two bars U1M2 and U1U3 which become ineffective structurally in case of critical damage to the end post L0U1.

APPENDIX B - SHALLOW TYPE TRUSSES

In order to maintain rational limitations on deflections of bridge structures, highway bridge specifications require that the ratio of depth to length of spans shall preferably be not less than 1/10 for trusses and 1/25 for girders. Since polygonal trusses are essentially rigid frames, whose resistance to flexure may be considered intermediate between girders and ordinary trusses, it appeared that shallower trusses than required for the conventional type might be utilized advantageously when polygonal trusses are involved. Accordingly, the high truss of Figure 5(a), with a depth to span ratio of about 1 to 7, was redesigned, for study and illustrative purposes only, into the relatively shallow truss shown in Figure 5(c) and having a depth ratio of about 1 to 10 (i.e., approximately two-thirds of that of the conventional type), but retaining the panel lengths and loadings of the higher truss. Normally, of course, the panel spacing would be modified to conform to the truss depth.

The redesign, based on the same procedure as that employed for the primary pentagonal truss, resulted in a redistribution of the metal between the chord and web systems, which, by coincidence, was nearly identical with the distribution of metal in the conventional truss shown in Figure 5(b). The estimated weights of this truss and its component systems are given in Table 4, Appendix C, together with those of the basic designs. The computed deflection for the shallow truss was found to be 4.4 inches at its midpoint, as compared with 3.08 inches for the high pentagonal truss and 3.38 inches for the prototype conventional truss.

The height of conventional trusses is an important factor in limiting the economical span of truss bridges. If the preceding observations are borne out by further investigation and test data, it would appear that practicable span lengths could be extended materially when polygonal trusses are considered for bridge construction. This possibility would be further enhanced by the fact that no auxiliary bracing is required for any of the web members of polygonal trusses, as is usually the case with the members of the longer span conventional trusses.

APPENDIX C - ESTIMATED WEIGHTS

Many factors, besides the weight of steel, influence the choice of type of bridge superstructure. Among these are relative design and reserve strength, rigidity, appearance, cost of fabrication and assembly and erection conditions. Nevertheless, comparative weights of material are an important guide in the final determination of the type of structure to be used. Accordingly, in Table 4 there are presented, comparatively, the estimated weights of the semi-rigid pentagonal trusses, Figures 5(a) and 5(c), and the actual weight of the prototype conventional truss of Figure 5(b). While the design criteria

and the characteristics of the two types of trusses are not directly comparable, the table is introduced here to serve merely as an indication of the relative weights of steel involved with particular attention to their distribution within these structures.

The estimated weights of the pentagonal trusses are based on designs conforming to A.A.S.H.O. Specifications and employing box type sections in the top and bottom chords, similar to those of the conventional truss, and tapered WF sections in their web members. As stated in the body of the paper, in the stress determination and design of the truss members it was assumed that all the chord joints were hinged. Nevertheless, as indicated in Figure 8, the chord joints were detailed to develop at least 25% carryover moment from the M-joints. In the conventional truss no consideration was given to the moments developed in its riveted joints.

In view of the above design procedures and relative rigidities of the trusses, it may be considered that the weight of steel in the pentagonal type compares favorably with those of the conventional design. If the alternate end panel arrangement of Figure 6 is incorporated in the design, the estimated saving shown in Table 3, Appendix A, would reduce the weight of the basic pentagonal truss from 211,000 to about 200,000 lbs.

It is interesting to note that the total truss weights for the high and shallow pentagonal trusses are substantially the same, although the relative weights of the chord and web systems have changed materially, resulting in huskier members throughout the lower structure. Since the weight of truss, for given span and panel lengths, is not likely to be a constant, there must be a height for which it is a minimum. It may be observed, however, that the higher truss, in which the weight of materials is more evenly distributed between the chord and web systems, appears slightly more economical than the shallow type. If this were proven true in a fully developed design it would generally conform to the theoretical derivations for economical weight of truss as given by J. A. L. Waddell, page 1183 in his monumental "Bridge Engineering" (1916) according to which the weight of truss (for parallel chords only) is a minimum when the chord weight equals the weight of the web system. However, realistic comparative weights would require a complete detailed design in which both the web and the chord systems are developed for an all-rigid pentagonal truss structure.

TABLE 1

STRESS SUMMARY FOR SEGMENT YM4

Bar	Length In.	Index stresses(b) Uniform Load: Shear Lbs.	Moment In-Lbs.	Axial Lbs.	Dead Load Stresses Index Stress (b) x K_D: Shear Kips	Moment In-Kips	Axial Kips	Uniform Live Load Stresses Index Stress (b)or(c) x K_L: Shear Kips	Moment In-Kips	Axial Kips
1	2	3	4	5	6	7	8	9	10	11
L2L4	332	-	-	5.64T	-	-	412.0 T	-	-	85.2 T
L4L6	332	-	-	7.54C	-	-	551.1 T	-	-	113.9 T
U3U5	336	-	-	6.9C	-	-	509.6 C	-	-	105.4 C
M4L4	210	1.904	400	1.0	139.1	29,216	73.0 T	30.8	6,466	15.1 T
U3M4	261	0.676	222	3.04	64.0	16,690	222.0 T	14.1	3,693	46.6 T
M4U5	302	0.566	172	1.94	41.5	12,526	141.7 C	9.2	2,772	32.3 C

TABLE 1 - (Continued)

Bar	Concentrated Live Load Stresses Index Stress (e) x K_C: Shear Kips	Moment In-Kips	Axial Kips	Live Load Impact Stresses (Uniform + Concentrated Live Loads) x K_I: Shear Kips	Moment In-Kips	Axial Kips	Maximum Total Stresses Dead + Live + Impact Stresses: Shear Kips	Moment In-Kips	Axial Kips
	12	13	14	15	16	17	18	19	20
L2L4	-	-	15.2 T	-	-	11.7 T	-	-	524.1 T
L4L6	-	-	20.1 T	-	-	15.6 T	-	-	700.7 T
U3U5	-	-	22.2 C	-	-	14.9 C	-	-	652.3 C
M4L4	1.92	402	15.4 T	4.37	918	4.1 T	176.2	37,002	107.6T
U3M4	0.80	230	11.3 T	2.00	525	8.0 T	81.0	21,138	289.9 T
M4U5	0.57	172	7.8 T	1.31	394	3.3 C	52.6	15,864	169.5 C

K_D = 73,050 lbs. = Panel dead load

K_L = 15,100 lbs. = Panel live load

K_C = 15,350 lbs. = Concentrated live load

K_I = 11.69% or 13.37% = Applicable impact percentages

T = Tension; C = Compression

Index stresses (b), (c), and (e) obtained from corresponding stress diagrams of Figure 7.

TABLE 2

DEAD LOAD DEFLECTION AT L10

Bar	Length L, inches	Area A, sq. in.	D.L. Stress: S, kips	$\frac{SL}{A}$	Unit Load Stress, u10: $\frac{}{A}$	$\frac{SLu10}{A}$
1	2	3	4	5	6	7
L0U1	385.6	45.22	-404.6	-3,450	-0.604	2,084
U1U3	338.1	36.36	-325.8	-3,030	-0.455	1,379
U3U5	335.7	47.38	-509.8	-3,610	-0.820	2,962
U5U7	334.0	53.88	-616.5	-3,820	-1.130	4,316
U7U9	332.7	60.38	-675.6	-3,720	-1.420	5,280
U9U11	332.1	62.72	-702.6	-3,720	-1.863	6,930
O.H.	--	--	--	--	--	18,725
L0L2	332.0	29.54	174.2	1,960	0.260	509
L2L4	332.0	36.44	412.0	3,750	0.659	2,475
L4L6	332.0	48.44	551.1	3,775	0.979	3,700
L6L8	332.0	54.76	633.0	3,840	1.272	4,880
L8L10	332.0	60.76	676.2	3,695	1.565	5,775
L10L10'	332.0	60.76	686.9	3,765	1.573	5,920
O.H.	--	--	--	--	--	18,297
M2L2	210.0	119.29	73.0	128	--	--
M4L4	210.0	92.18	73.0	166	--	--
M6L6	222.0	69.48	73.0	233	--	--
M8L8	226.0	57.75	73.0	288	--	--
M10L10	240.0	35.95	73.0	487	1.000	487
O.H.	--	--	--	--	--	--
U1M2	216.0	92.18	306.7	719	0.435	313
M2U3	261.0	69.48	-235.9	-887	-0.490	435
U3M4	261.0	69.48	222.0	835	0.430	359
M4U5	302.0	55.33	-141.7	-773	-0.440	340
U5M6	292.0	48.75	159.2	954	0.430	410
M6U7	322.0	33.70	- 78.9	-753	-0.440	331
U7M8	317.0	41.05	109.6	847	0.460	390
M8U9	336.0	33.63	- 21.9	-219	-0.470	103
U9M10	325.5	30.72	62.1	658	0.585	385
M10U11	332.4	27.26	- 21.9	-256	0.580	-150
O.H.	--	--	--	--	--	2,621

Total = 89,256

At L10, deflection $D = \frac{89,256 \times 1,000}{29,000,000} = 3.08$ in.

Notes: 1. Deflections shown are due to axial stresses only.
2. The stresses u10 due to unit load at L10 are obtained in the same manner as actual stresses in the truss members.
3. O.H. denotes 'other half' of truss for which only the summaries for each group of members of column 7 are given for brevity.

TABLE 3

BASIC AND ALTERNATE END PANEL

Bar	Maximum Total Stresses				Estimated Weights(a)		
	Basic Design			Alternate Design	Basic	Alternate	Savings
	Shear Kips	Moment In.-Kips	Axial Kips	Axial Kips	Lbs.	Lbs.	Lbs.
L0L2	--	--	221.9 T	524.1 T	6,440	6,440	--
L0U1	--	--	515.4(b)C	212.1(b)C	11,350	8,650	2,700
U1U3	--	--	419.4 C	267.5 C	8,360	7,660	700
U1M2	173.4	37,424	393.5 T	228.5 T)			
M2L2	297.2	62,433	107.1 T	114.6 T)	28,500	11,500	17,000
M2U3	95.9	25,009	226.7 C	435.8 C)			
L0M2	--	--	--	526.4 C	--	8,640	(-8,640)
Total	--	--	--	--	54,650	42,850	11,800

(a) For one truss.
(b) Exclusive of end portal stresses.

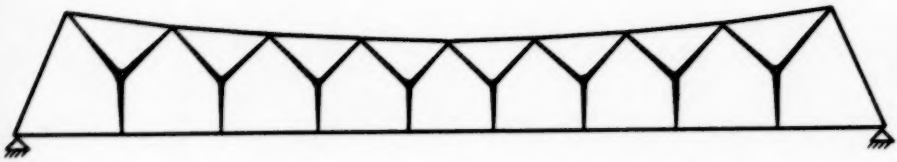
T = Tension
C = Compression

TABLE 4

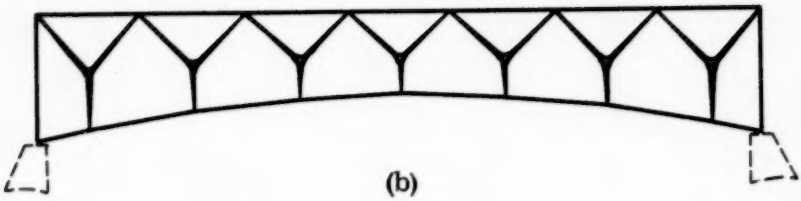
ESTIMATED WEIGHTS - ONE TRUSS ONLY

Truss System	PENTAGONAL TRUSSES				CONVENTIONAL TRUSS	
	High, Fig. 5 (a)		Shallow, Fig. 5(c)		Truss, Fig. 5 (b)	
	Weight (Lbs.)	%	Weight (Lbs.)	%	Weight (Lbs.)	%
Top Chord(b)	67,300	31.9	85,000	38.3	86,200	41.5
Bottom Chord	44,000(c)	20.8	67,900(c)	30.6	74,400(d)	35.8
Web System	100,000	47.3	69,000	31.1	47,200(e)	22.7
Total Weight(a)	211,300	100.0	221,900	100.0	207,800	100.0

(a) Pentagonal trusses shop welded, field riveted; conventional truss shop and field riveted.
(b) Includes top chord gussets.
(c) Exclusive of bottom chord gussets.
(d) Includes bottom chord gussets.
(e) Exclusive of gussets.

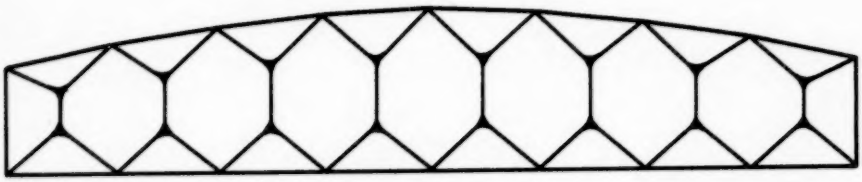


(a)

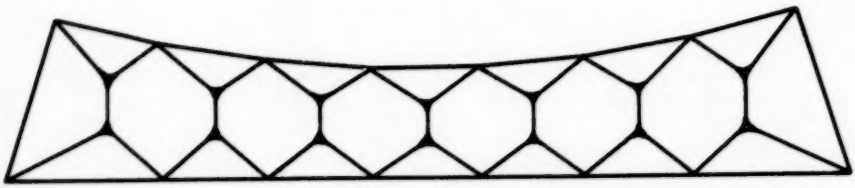


(b)

Fig. 1. Pentagonal Trusses

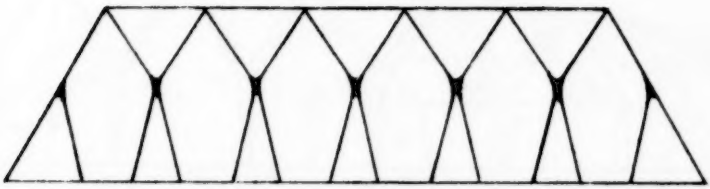


(a)

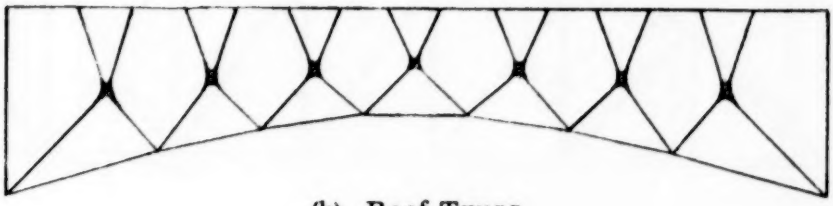


(b)

Fig. 2. Hexagonal Trusses



(a) Through Truss



(b) Roof Truss

Fig. 3. Subdivided Panel Trusses

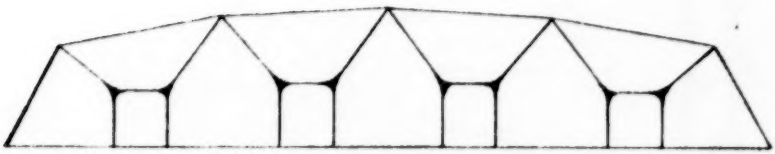
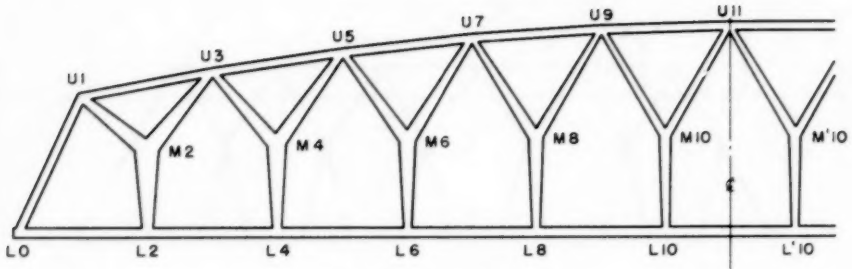
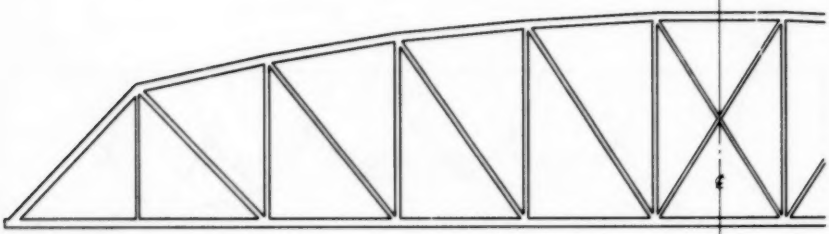


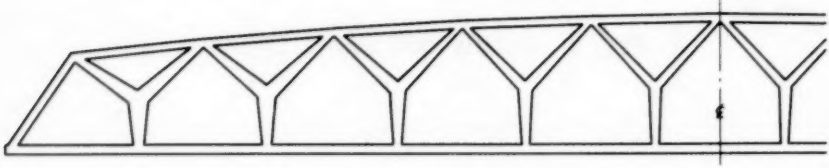
Fig. 4. Double Stem Pentagonal Truss



(a) High Pentagonal Truss



(b) Conventional Truss



(c) Shallow Pentagonal Truss

Fig. 5.

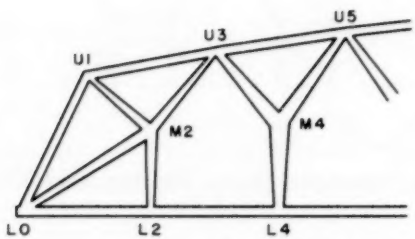
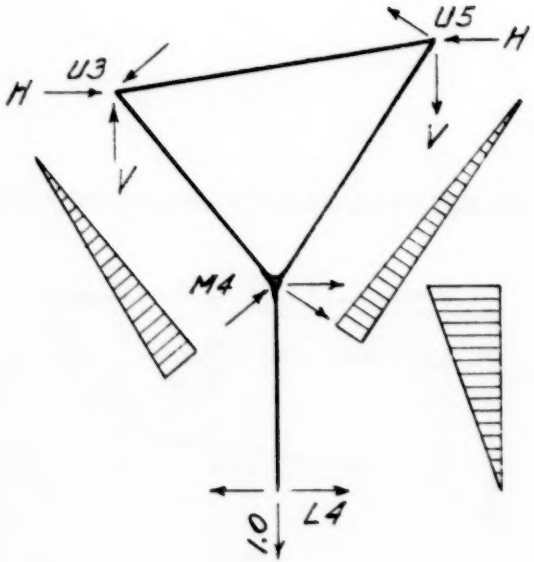
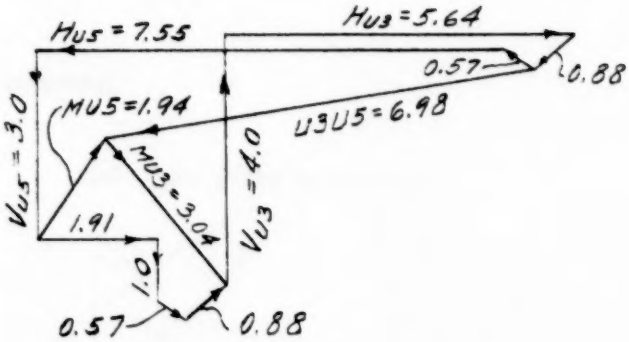


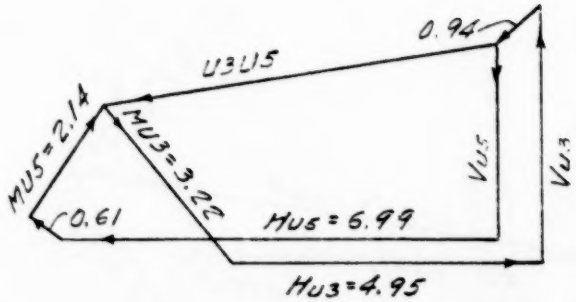
Fig. 6. Alternate End Panel (For Fig. 5a)



(a) Moments, Shears and Hinge Loads



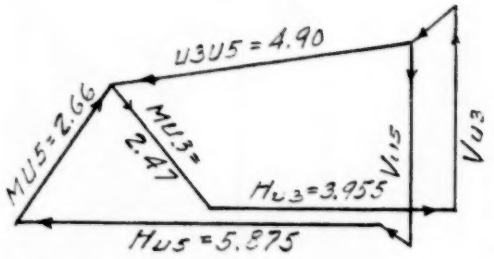
(b) Unit Loads at all L-Joints



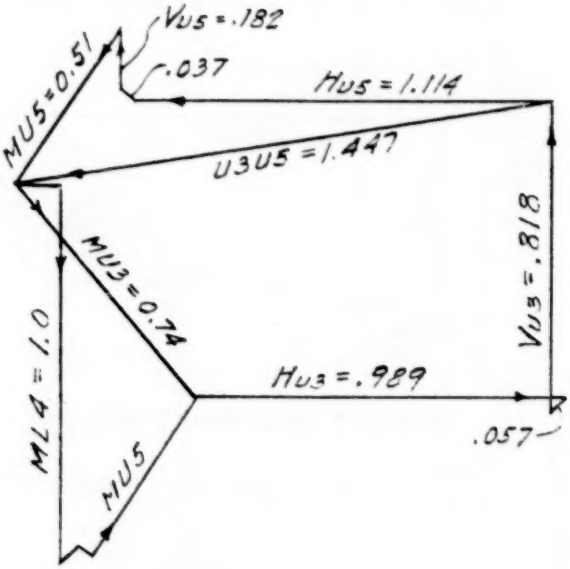
(c) Unit Loads at Joints L4 to L'2 incl.

Fig. 7. Stress Analysis, Segment YM4

828-17



(d) Unit Loads at Joints L6 to L'2 incl.



(e) Unit Load at L4 only

Fig. 7. (Continued)

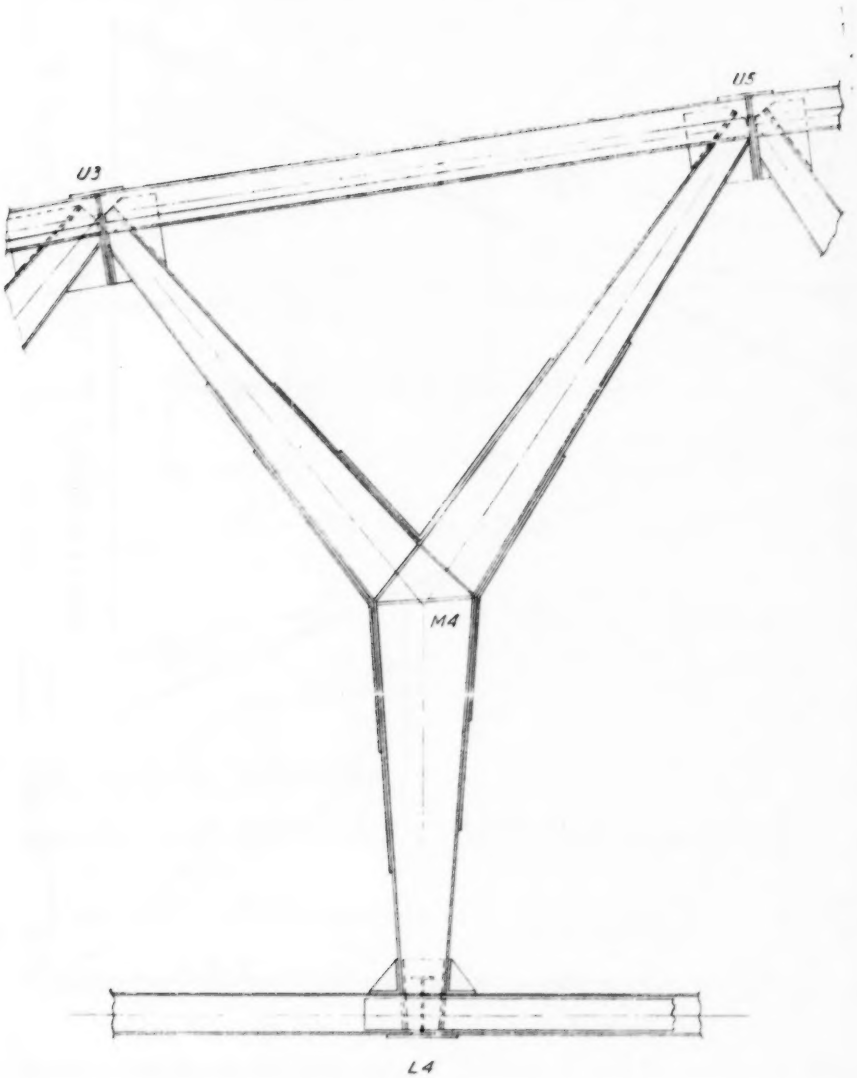
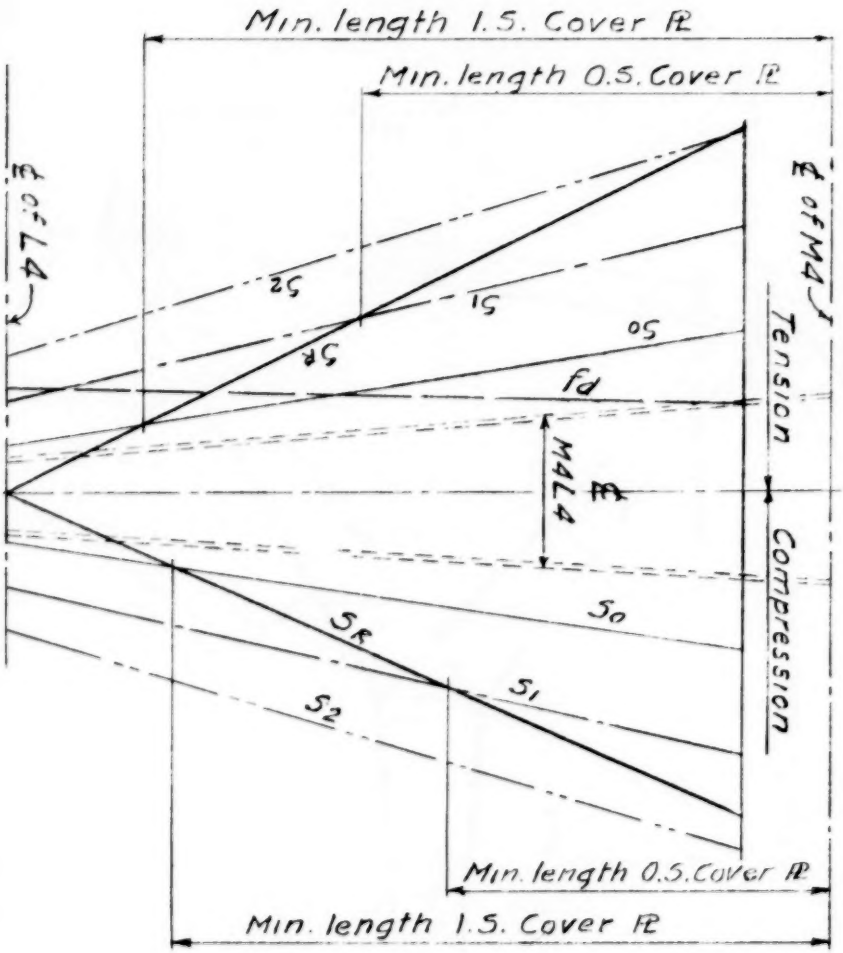


Fig. 8. Segment YM4 - H-type Sections



LEGEND:

S_R = Required S.M. after direct stress

S_0 = Available S.M., no cover pls.

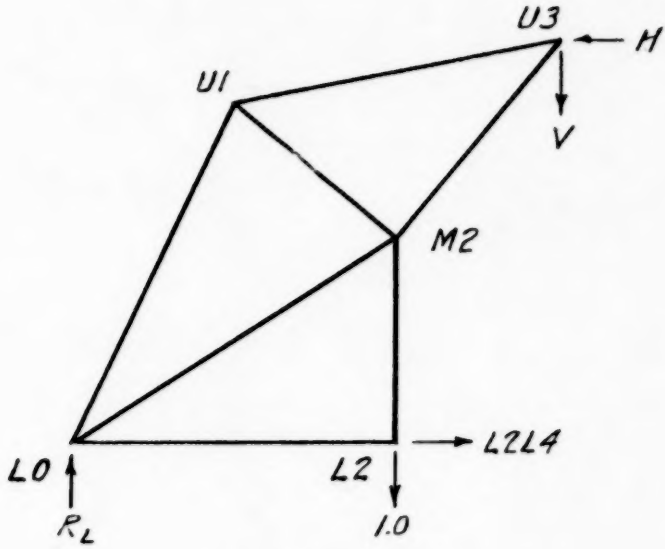
S_1 = " " , one cover pl.

S_2 = " " , two cover pls

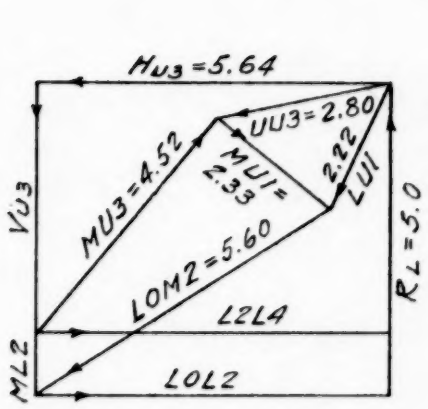
f_d = Direct stress variation.

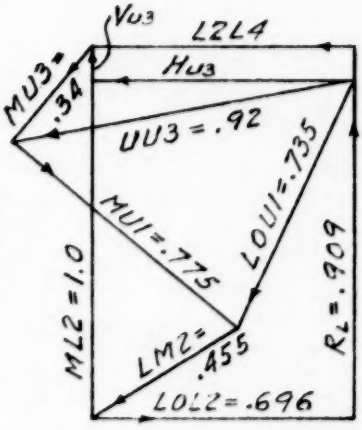
S.M. = Section Modulus.

Fig. 9. Length of Cover Plates



(a) Hinge Loads





(b) Unit Loads at all L-Joints (c) Unit Load at Joint L2 only

Fig. 10. Modified End Panel

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